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Exact noncommutative solitons in p-adic strings and BSFT

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Abstract: The tachyon field of p-adic string theory is made noncommutative by replacing ordinary products with noncommutative products in its exact effective action. The same is done for the boundary string field theory, treated as the $p \to 1$ limit of the p-adic string. Solitonic lumps corresponding to D-branes are obtained for all values of the noncommutative parameter θ. This is in contrast to usual scalar field theories in which the noncommutative solitons do not persist below a critical value of θ. As θ varies from zero to infinity, the solution interpolates smoothly between the soliton of the p-adic theory (respectively BSFT) to the noncommutative soliton.

Keywords: Solitons Monopoles and Instantons, p-branes, String Field Theory, Non-Commutative Geometry.

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Contents

1. Introduction

The progress in our understanding of the tachyon field localized on an unstable D-brane, led by the pioneering work of Sen (see [1] and references therein), has opened a new window into the dynamics of string theory. However, the full string theory being intractible, one hopes to gain insight into its qualitative features by studying simpler models which share its essential properties. The p-adic string theory, introduced in [2] and studied further in [3]–[10] (see [11] for a review), is one such model on which one has considerable analytic control. Indeed, in [12] it was shown that the tree level effective action of the p-adic tachyon, the expression for which is known exactly, provides an explicit realization of the Sen conjectures. This, and other properties, were explored further in [13]–[19]. Although the p-adic string itself is an exotic object, the spacetime it describes is the familiar one.[1] Therefore, field theoretical tools can be applied profitably.

Moreover, it turns out [21] that a limit in which p tends to one, approximates the boundary string field theory (BSFT) description of the usual bosonic string [22, 23]. More precisely, upto two derivatives, the effective action of the tachyon can be computed exactly in BSFT. This matches with the $p \to 1$ limit of the tachyon of the p-adic string. Many of the features of the p-adic theory, e.g., the existence of gaussian lump solitons, survive this limit. The results obtained in BSFT are in accordance with the conjectures of Sen [24, 25].

In spite of these progress, p-adic string still remains an enigmatic entity. We certainly lack an understanding of p-adic closed string. Even open string modes, other than the tachyon, are very poorly understood. It is therefore almost impossible to address questions about p-adic string in non-trivial backgrounds, due either to closed strings, like curved spacetime, or to open strings, like an electromagnetic field. It should be recalled that

[1] For a different, perhaps more exotic, type of p-adic string, see [20].

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thanks to refs. [7], we do have a 'worldsheet' action for the p-adic string. However, the 'worldsheet' being a tree, i.e., an infinite Bethe lattice, the interpretation of it is difficult. Consequently the worldsheet approach, so fruitful for usual strings, is yet to be exploited to our advantage.

We do, however, know of one background, namely a constant background of the antisymmetric second rank tensor field B, that leads to a simple modification of the spacetime effective action. One can use the same form of the action, but use a noncommutative product, instead of the ordinary product, in multiplying fields. There is a vast literature on the subject, see ref. [26] for a review and references. In particular, it leads to a great simplification in undertstanding tachyon condensation. Generic noncommutative scalar field theories admit solitonic lump solutions [27], which can be identified with D-branes [28] (see also the review [29]).

In this paper, we propose to modify the spacetime action of the p-adic tachyon by replacing the ordinary product of fields by the noncommutative Moyal product. This will be our definition of the noncommutative p-adic tachyon. We will study the resulting equations of motion and find gaussian lump solutions for *all* values of the noncommutative parameter θ. These are shown to interpolate from the usual p-adic solitons to the soliton found in ref. [27] in the limit of infinitely large noncommutativity. Next we consider the $p \to 1$ limit [21], which is known to be the bosonic string in the boundary string field theory (BSFT) formalism [22]–[25]. Exact noncommutative soltions are obtained for a noncommutative deformation [30, 31] of this theory. We will also comment on multisolitons.

In the following, we sometimes use the economical, if somewhat confusing, abbreviations p-tachyon, p-string and p-soliton for the tachyon, string and soliton respectively of the p-adic string theory. In the $p \to 1$ limit, these naturally connect to the usual tachyon, etc!

The solitons of the noncommutative p-adic string theory were presented in the conference *p-Adic MathPhys 2003* held at the Steklov Institute, Moscow [32].

2. Review of the p-adic tachyon

In p-adic string theory *all* tree level amplitudes involving tachyons in the external states can be computed. This makes it possible to write the exact effective action for the p-tachyon field. It was computed in [4] and is described by the lagrangian

$$\mathcal{L}_p = -\frac{1}{2}\varphi\, p^{-\frac{1}{2}\Box}\varphi + \frac{1}{p+1}\,\varphi^{p+1}\,. \tag{2.1}$$

Although this was arrived at by computing Koba-Nielsen amplitudes in the p-adic theory, which is meaningful and possible only for a prime p, the final spacetime action makes sense for all integer values of p. The potential has a local minimum and two (respectively one) local maxima for odd (respectively even) integer p. There are always runaway pathological singularities.

The equation of motion following from the above is

$$p^{-\frac{1}{2}\Box}\varphi = \varphi^p\,. \tag{2.2}$$

Apart from the trivial constant solutions for $\varphi = 0, 1$, this admits soliton solutions. In fact, the equations separate in the arguments and for any (spatial) direction y, we get

$$\varphi(y) = p^{1/2(p-1)} \exp\left(-\frac{p-1}{2p\ln p}\, y^2\right), \tag{2.3}$$

a gaussian lump whose amplitude and spread are correlated. While the solutions were already found by the authors of ref. [4], these were identified as the D-branes of the p-string theory in [12].

More specifically, considering a lump along the last $25 - m$ dimensions, one has a configuration in which energy is localized in codimension $25 - m$. This is to be identified with the D-m-brane of the p-string theory. There are branes for all values $m = 0, 1, \ldots, 25$ satisfying descent relations conjectured by Sen [1]. In particular, the ratios of their tensions match that of the energies of the corresponding solitons. It was also shown that one gets a consistent truncation of the worldvolume theory on a D-m-brane by keeping only the tachyon mode, the resulting theory is exactly of the form of the parent theory in $(m + 1)$ dimensions.

3. A noncommutative deformation of the p-adic tachyon

In this section, we will study the p-adic string in a nontrivial background, namely the one produced by a constant two-form antisymmetric tensor field B. In usual string theory, this has the effect that spatial coordinates no longer commute, instead

$$[x^i, x^j] = i\theta\epsilon^{ij}, \tag{3.1}$$

where, θ is related to the constant value of B_{ij}. For simplicity, let us restrict our attention to the minimal case in which two spatial directions, say x^1 and x^2 fail to commute. An equivalent description of the Physics is through the use of commuting coordinates, but while multiplying fields, which are dependent on x^1 and x^2, one uses the Moyal star product

$$f(x^1, x^2) \star g(x^1, x^2) = f(x^1, x^2) \exp\left(\frac{i}{2}\theta\epsilon^{ij}\overleftarrow{\partial}_i\overrightarrow{\partial}_j\right) g(x^1, x^2), \tag{3.2}$$

instead of the ordinary pointwise multiplication. Alternatively, f, g may be thought of as operator valued functions through the well-known Moyal-Weyl correspondence [26, 29].

Ordinary commutative field theories may be deformed by the use of the Moyal star product. These are known to exhibit rather interesting properties, both perturbative as well as nonperturbative. For example, nontrivial soliton solutions were found [27] in a noncommutative scalar field theory with generic polynomial potential in the limit of infinite noncommutativity: $\theta \to \infty$. These solutions owe their existence to the infinite number of derivatives that appear in the star product. They are essentially the solutions of the equation

$$\phi \star \phi \sim \phi \tag{3.3}$$

defining projectors. The simplest of the solitons is a gaussian lump whose width and amplitude are fixed:

$$\phi(x^1,x^2) = 2\exp\left(-\frac{(x^1)^2+(x^2)^2}{\theta}\right). \tag{3.4}$$

These solitons become unstable at finite values of θ.

On the other hand, the effective action (2.1) of the p-tachyon field already contains an infinite number of derivatives. Moreover, there are gaussian lump solutions of the equations of motion. This leads to the hope that a noncommutative deformation of the p-tachyon may admit solitons that are stable at all values of the noncommutative parameter θ. We will see that this indeed turns out to be the case for the gaussian soliton.

In order to make a noncommutative deformation, we shall follow the standard practice and replace ordinary products of the p-tachyon field φ by the star product (3.2) in the action (2.1). This gives the action for the noncommutative p-tachyon:

$$\mathcal{L}_p^{NC} = -\frac{1}{2}\varphi\star p^{-\frac{1}{2}\Box}\varphi + \frac{1}{p+1}(\star\varphi)^{p+1}, \tag{3.5}$$

where we have used an obvious shorthand notation for the $(p+1)$-fold noncommutative product of φ. For us, the action (3.5) *defines* the noncommutative p-tachyon. It will, however, be desirable, using the results of [7], to develop a worldsheet understanding of this deformation along the lines of [33].

The equation of motion, as usual, is the noncommutative variant of (2.2):

$$p^{-\frac{1}{2}\Box}\varphi = (\star\varphi)^p. \tag{3.6}$$

We will only be interested in the part dependent on x^1 and x^2. It will, therefore, suffice to restrict our attention to these directions only. Hence $\varphi=\varphi(x^1,x^2)$ and $\Box=\partial_1^2+\partial_2^2$ for us.

The trivial solutions, $\varphi=0,1$ describing constant configurations, are obviously still solutions of (3.6). More interestingly, there is a nontrivial solution. This is a gaussian solitonic lump and may be obtained by Fourier transformation on a gaussian ansatz. To this end, let us note that the n-fold star product of the gaussian

$$g(\mathbf{x}) = A^2\exp\left(-ar^2\right), \qquad \mathbf{x}=(x^1,x^2), \qquad r^2=(x^1)^2+(x^2)^2, \tag{3.7}$$

is again a gaussian albeit with modified width and amplitude:

$$(\star g)^n(\mathbf{x}) = \frac{A^{2n}}{\sum_{i=0}^{\lfloor n/2\rfloor}\binom{n}{2i}(a\theta)^{2i}}\exp\left[-\frac{\sum_{i=0}^{\lfloor (n-1)/2\rfloor}\binom{n}{2i+1}(a\theta)^{2i}}{\sum_{i=0}^{\lfloor n/2\rfloor}\binom{n}{2i}(a\theta)^{2i}}ar^2\right]. \tag{3.8}$$

It is easy to obtain the above by induction. The differential operator on the LHS of (3.6) also modifies the width and amplitude of the gaussian (3.7). Thus one gets a solution to the equation of motion by equating the width and amplitude of the gaussian on both sides of (3.6).

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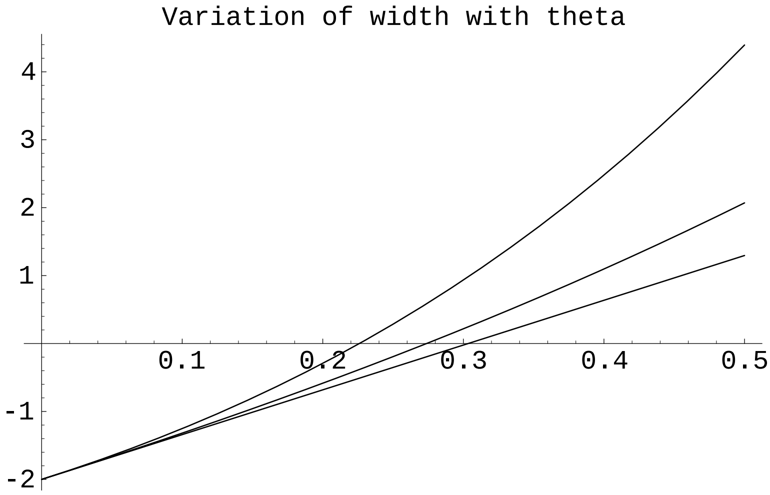

Figure 1: Polynomial (3.9) for $p = 3$ for different θ (increasing upwards).

The width a is determined by a polynomial of degree p:

$$\sum_{i=0}^{\lfloor p/2 \rfloor} \binom{p}{2i} (a\theta)^{2i} - (1 - 2a \ln p) \sum_{i=0}^{\lfloor (p-1)/2 \rfloor} \binom{p}{2i+1} (a\theta)^{2i} = 0\,. \tag{3.9}$$

For an odd integer p, one is guaranteed to have one real root. (For $p = 2$, the roots turn out to be real, the positive root being the relevant one.) While it is not possible to determine a explicitly as a function of θ for most p, we can check that the known limits are recovered. In the commutative limit $\theta = 0$, the polynomial reduces to a linear equation in a, the solution of which is the result $a = \frac{p-1}{2p \ln p}$ in eq. (2.3). On the other hand, in the limit of infinite noncommutativity $\theta \to \infty$, one cannot naively keep only the term of highest degree in a. Using the fact that $a \sim 1/\theta$ in this case[27], this term is actually non-leading. It is easy to check that the constant of proportionality is one, thus one gets the value in eq. (3.4). The polynomial (3.9) for $p = 3$ is plotted in figure 1 for different values of θ. For increasing θ, the root moves towards the origin, as expected.

The amplitude likewise is determined in terms of a from

$$A = \left[\sum_{i=0}^{\lfloor (p-1)/2 \rfloor} \binom{p}{2i+1} (a\theta)^{2i} \right]^{1/2(p-1)}\,, \tag{3.10}$$

which, again, interpolates between the values $A = p^{1/2(p-1)}$ and $A = \sqrt{2}$ in the commutative and noncommutative limits respectively. (In the case of $p = 2$, the amplitude is a constant $A = \sqrt{2}$ independent of θ.)

In summary, we get, for all values of θ, a gaussian lump solution which interpolates smoothly between the soliton (2.3) of the p-string theory on one hand and the noncommutative GMS soliton (3.4) on the other hand.

It is easy to check that the total energy (see [15, 16] for the definition of energy in p-string theory) of the p-soliton

$$E = \frac{p-1}{2(p+1)} \int d^2x \, (\star g)^{p+1} = \frac{\pi(p-1)A^{2p+2}}{2(p+1)} \left[a \sum_{i=0}^{\lfloor p/2 \rfloor} \binom{p+1}{2i+1} (a\theta)^{2i} \right]^{-1} , \tag{3.11}$$

with g given by (3.7), (3.9) and (3.10) is a function of θ, with $E \sim \mathcal{O}(1)$ for $\theta = 0$ and $E \sim \theta$ for large values of the noncommutative parameter.

4. The $p \to 1$ limit and BSFT

Let us consider the $p \to 1$ limit now. In this limit, the lagrangian (2.1) of the p-string theory reduces to [21][2]

$$\mathcal{L}_{p\to 1} = \frac{1}{2}\varphi \, \Box \varphi + \frac{1}{2}\varphi^2 \left(\ln \varphi^2 - 1 \right) . \tag{4.1}$$

Curiously, this is exactly the action of the usual bosonic string, truncated to two derivatives, in the boundary string field theory (BSFT) [22, 23] approach.

A noncommutative deformation of this theory is described by the lagrangian

$$\mathcal{L}^{NC}_{p\to 1} = \frac{1}{2}\varphi \star \Box \, \varphi + \frac{1}{2}\varphi \star \varphi \star \left(\ln_\star (\varphi \star \varphi) - 1 \right) \tag{4.2}$$

yielding the equation of motion

$$\Box \, \varphi + 2\varphi \star \ln_\star \varphi = 0 . \tag{4.3}$$

In the context of BSFT, the spacetime noncommutativity of the tachyon was derived in refs. [30, 31] by taking into account the effect of the constant B-field exactly in the world-sheet Green's function of the fields $X^{1,2}$.

Since the commutative theory admits a gaussian soliton, it is natural to try the ansatz (3.7) for the solution of (4.3). However, the second term poses some problem. Neither do we have an expression for the star-deformed logarithm of an ordinary exponential function, nor does the polynomial (3.9) seem to have a meaningful $p \to 1$ limit. Nevertheless, we can go back to the eq. (3.6) and take the $p \to 1$ limit on the ansatz (3.7). The first term is:

$$\lim_{\epsilon \to 0} \left[\frac{1}{\epsilon}(1+\epsilon)^{-\frac{1}{2}\Box} - \frac{1}{\epsilon} \right] g(\mathbf{x}) = -\frac{1}{2}\Box \, g(\mathbf{x}) = 2a(1 - ar^2) \, A^2 e^{-ar^2} , \tag{4.4}$$

the same as in the commutative case. Now, let us note the pattern of terms in the n-fold star product of the gaussian in (3.8) — the coefficients of $a\theta$ are determined by those in the expansion of $(1+x)^n$. Ignorning the limit on the sum, as is appropriate for the non-integer power $1+\epsilon$, and dropping terms of order ϵ^2 or higher, we write:

$$(\star g(\mathbf{x}))^{1+\epsilon} = \frac{A^{2(1+\epsilon)}}{1+\epsilon \left(\frac{(a\theta)^2}{1.2} + \frac{(a\theta)^4}{3.4} + \cdots \right)} \exp \left[-ar^2 \left(\frac{1+\epsilon \left(1 - \frac{(a\theta)^2}{2.3} - \frac{(a\theta)^4}{4.5} - \cdots \right)}{1+\epsilon \left(\frac{(a\theta)^2}{1.2} + \frac{(a\theta)^4}{3.4} + \cdots \right)} \right) \right] . \tag{4.5}$$

[2] Curiously, Spokoiny[6] considered this limit and speculated on its relation to the usual string theory already in the early days of p-adic string theory!

From this we get the term in the RHS of the equation of motion:

$$\lim_{\epsilon\to 0}\frac{1}{\epsilon}\left[(\star g(\mathbf{x}))^{1+\epsilon}-g(\mathbf{x})\right]= \tag{4.6}$$
$$=\left[\left(2\ln A-\frac{(a\theta)^2}{1.2}-\frac{(a\theta)^4}{3.4}-\cdots\right)-2ar^2\left(\frac{1}{2}-\frac{(a\theta)^2}{1.3}-\frac{(a\theta)^4}{3.5}-\cdots\right)\right]A^2e^{-ar^2}.$$

Comparing eqs. (4.4) and (4.6), we find that the amplitude is determined by the transcendental equation:

$$2a=1-2\left(\frac{(a\theta)^2}{1.3}+\frac{(a\theta)^4}{3.5}+\frac{(a\theta)^6}{5.7}+\cdots\right)=\frac{1-(a\theta)^2}{2a\theta}\ln\left(\frac{1+a\theta}{1-a\theta}\right), \tag{4.7}$$

which interpolates between the width $a=1/2$ of the BSFT soliton and that of the non-commutative GMS soliton $a=1/\theta$. The amplitude is determined in terms of a as:

$$2\ln A=2a+\frac{(a\theta)^2}{1.2}+\frac{(a\theta)^4}{3.4}+\frac{(a\theta)^6}{5.6}+\cdots=\frac{1+a\theta}{2a\theta}\ln(1+a\theta)-\frac{1-a\theta}{2a\theta}\ln(1-a\theta). \tag{4.8}$$

This is also a smooth function of θ connecting the BSFT soliton with $A=\sqrt{e}$ to the GMS one with $A=\sqrt{2}$.

The total energy of the gaussian lump $E=\frac{1}{2}\int d^2x\, g\star g=\pi A^4/2a$ is again a function of θ through eqs. (4.7) and (4.8), varying from $E_{\theta=0}=2\pi e^2$ to $E_{\theta\to\infty}=4\pi\theta$.

As an aside, let us attempt a naive power series expansion for $g(\mathbf{x})\star\ln_\star g(\mathbf{x})$, which gives the series:

$$\begin{aligned}A^2\Bigg[&-\zeta(1)\exp(-ar^2)+\frac{1}{2}\frac{A^2}{(1+a^2\theta^2)}\exp\left(-\frac{2ar^2}{1+a^2\theta^2}\right)+\\&+\frac{1}{24}\frac{A^4}{(1+a^2\theta^2)(1+3a^2\theta^2)}\exp\left(-\frac{3+a^2\theta^2}{1+3a^2\theta^2}ar^2\right)-\\&-\frac{1}{72}\frac{A^6}{(1+3a^2\theta^2)(1+6a^2\theta^2+a^4\theta^4)}\exp\left(-\frac{4+4a^2\theta^2}{1+6a^2\theta^2+a^4\theta^4}ar^2\right)+\cdots\Bigg].\end{aligned} \tag{4.9}$$

The above, in which we have used the ζ-function regularization for the infinite sums of powers of the natural numbers, should be contrasted with the naive series expression of its commutative analogue:

$$A^2e^{-ar^2}\ln\left(A^2e^{-ar^2}\right)=A^2\left[-\zeta(1)e^{-ar^2}+\frac{1}{2}e^{-2ar^2}+\frac{1}{24}e^{-3ar^2}-\frac{1}{72}e^{-4ar^2}+\cdots\right]. \tag{4.10}$$

For consistency, the series (4.9) must sum to the RHS of eq. (4.6). This identity gives the $\star$-logarithm of an ordinary exponential function.

5. Comments on multisolitons

In addition to the gaussian (3.4), the eq. (3.3) admits other solutions which can be interpreted as multisolitons [27]. However, these are unstable and reduce to (3.4) at any finite θ. An interesting class of multisolitons were studied in [34]. Let

$$f_{w_1w_2}=\exp\left(-\frac{1}{\theta}(\bar z-\bar w_1)(z-w_2)\right), \tag{5.1}$$

where $z = x^1 + ix^2$ and we will refer to (w_1, w_2) as the two centres. The configuration

$$\Pi = \sum_{i,j=1}^{n} A_{ij} f_{w_i w_j} = \sum_{i,j} A_{ij} e^{-(\bar{z}-\bar{w}_i)(z-w_j)/\theta} \tag{5.2}$$

solves $\Pi \star \Pi = \Pi$ (for any value of θ) for specific choices of A_{ij}, and hence the equations of motion (3.3) of a noncommutative scalar field theory at $\theta \to \infty$. By the Moyal-Weyl correspondence $f_{w_i w_j} \sim |w_i\rangle\langle w_j|$, where $|w\rangle \sim e^{w a^\dagger}|0\rangle$ is a *coherent state*. The amplitudes of the multisoliton (5.2) are determined in terms of the inner products of these states: $||A_{ij}|| \sim ||\langle w_i|w_j\rangle||^{-1}$. To be more precise, eq. (5.2) (given in terms of (5.1)), is the multisoliton for distinct values of w_i well-separated on the complex plane. When $w_1, w_2, \cdots, w_\ell$ approach the same value, say, w, the solution is in terms of projection operators in an a-invariant subspace of the Hilbert space spanned by $\{|w\rangle, a^\dagger|w\rangle, \cdots, (a^\dagger)^{\ell-1}|w\rangle\}$. The multisolitons of [34] have a moduli space that survive to order $\mathcal{O}(1/\theta)$.

Inspired by this, let us define the more general two-centred gaussian

$$f_{w_i w_j}(a_{ij}, A_{ij}) = A_{ij} \exp\left(-a_{ij}(\bar{z}-\bar{w}_i)(z-w_j)\right), \tag{5.3}$$

where a_{ij} and A_{ij} are real. We will try a multisoliton ansatz in terms of (5.3) for the equation of motion (3.6) of the p-tachyon. It is straightforward to check that:

$$\begin{aligned}
f_{w_1 w_2}(a_{12}, A_{12}) \star f_{w_3 w_4}(a_{34}, A_{34}) &= f_{\tilde{u}\tilde{v}}(\tilde{a}, \tilde{A}), \\
\tilde{a} &= \frac{a_{12}+a_{34}}{1+\theta^2 a_{12}a_{34}}, \\
\tilde{A} &= \frac{A_{12}A_{34}}{1+\theta^2 a_{12}a_{34}} \exp\left\{-\frac{a_{12}a_{34}}{a_{12}+a_{34}}(\bar{w}_1-\bar{w}_3)(w_2-w_4)\right\}, \\
\tilde{u} &= \frac{a_{12}(1+a_{34}\theta)}{a_{12}+a_{34}} w_1 + \frac{a_{34}(1-a_{12}\theta)}{a_{12}+a_{34}} w_3, \\
\tilde{v} &= \frac{a_{12}(1-a_{34}\theta)}{a_{12}+a_{34}} w_2 + \frac{a_{34}(1+a_{12}\theta)}{a_{12}+a_{34}} w_4.
\end{aligned} \tag{5.4}$$

Thus, not only does the width and amplitude get modified as in (3.8), the centres also get shifted. Thus, the $\star$-product of $\sum_{ij} f_{w_i w_j}(a_{ij}, A_{ij})$ with itself generates new terms.

The LHS of (3.6), on the other hand, modifies only the width and amplitude:[3]

$$e^{-\frac{1}{2}\ln p\,\Box} f_{w_i w_j}(a_{ij}, A_{ij}) = f_{w_i w_j}\left(\frac{a_{ij}}{1+2a_{ij}\ln p}, \frac{A_{ij}}{1+2a_{ij}\ln p}\right), \tag{5.5}$$

but leaves the centres unchanged. Thus the multisolitons of the type found in ref. [34] do not solve the equation of motion (3.6) of the p-tachyon. However, since the $p \to 1$ limit is the usual bosonic strings, one would expect to have multisolitons in this limit. It would be interesting to see how that happens.

The centres do not shift in the eq. (5.4) if $a_{ij} = 1/\theta$, or if $w_1 = w_3$ and $w_2 = w_4$. In both cases, $\tilde{u} = w_1$ and $\tilde{v} = w_4$ and hence $f_{w_1 w_2} \star f_{w_3 w_4} \sim f_{w_1 w_4}$. The former case is what we expect from the Moyal-Weyl correspondence. In particular, this is true in the

[3] One may say the differential operator maps operators of the form (5.3) to a different quantum mechanical Hilbert space with a modified value of the Planck constant.

$\theta \to \infty$ limit recovering the multisoliton of [34]. In the second case, we find that $f_{ww'}(a, A)$ solves the equation of motion (3.6) of the p-tachyon with a and A given by (3.9) and (3.10) respectively. However, this solution is complex, unless $w = w'$, in which case we find a simple generalization of the gaussian solution of section 3 centred around the point w in the z-plane. This corresponds to the moduli due to the translation zero-modes of the solitons (found in refs. [8, 12] for (2.3) at $\theta = 0$).

6. Concluding remarks

We obtained gaussian lump solutions to the equations of motion of a noncommutative deformation of the tachyon of the p-adic string theory for all values of the noncommutative parameter θ. It is shown that this one parameter family of solutions survives the $p \to 1$ limit, in which the p-adic string approximates the BSFT description of the usual bosonic string. The solutions interpolate smoothly between the soliton of the commutative p-adic tachyon (respectively BSFT) and the GMS soliton in the limit of infinite noncommutativity. This is unlike other exact solutions obtained in field theories of scalars and gauge fields, see e.g. [35], which do not have a smooth commutative limit.

It should be mentioned that the similarity between the solitons of the p-string theory and the noncommmutative field theories have been noticed in ref. [36]. In particular, it was pointed out that the expression (3.4) for $\theta = 4 \ln 2$ is identical to solitonic 2-brane of the 2-adic string theory. However, this seems to be a fortuitous coincidence without any particular significance. Indeed, the GMS soliton (3.4) is a solution near $\theta \to \infty$.

We have introduced noncommutativity in the p-adic string theory by replacing ordinary products with the Moyal star product in the spacetime effective action of its tachyon. This is admittedly rather ad hoc. In usual string theory this deformation is due to a constant background of the second rank antisymmetric tensor field B, which alters the worldsheet properties of the fields $X^{1,2}$. This can be taken into account exactly in the (worldsheet) Green's functions of these fields. The 'worldsheet' of the p-adic string is an infinite Bethe lattice [7]. It would be interesting to see if the lattice worldsheet action admits a deformation by the B-field that induces the spacetime noncommutativity. We hope to report on this in future [37].

Ref. [33] noted the similarity between the spacetime noncommutativity arising out of a B-field and the methods of deformation quantization. It would nice if noncommutative p-tachyon can be used in the quantization of its nonlocal field theory.

Finally, the p-adic string seems to have many of the features in common with the usual string theory. Moreover, there are direct connections through the adelic relations [3] and the $p \to 1$ limit [21]. For these reasons, it may be worthwhile to develop a better understanding of the p-adic string theory. As of now, we only know some properties of its D-branes in flat space. Even there the tension of the D-brane has not been computed from a worldsheet description. This, as well as other problems, e.g., the B-field background that we studied in this paper, call for a proper understanding of the closed p-adic string.

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